Consider the polar equation $r = \dfrac{60}{3-7\sin\theta}$. $= \dfrac{20}{1-\frac{7}{3}\sin\theta}$ 2 $e = \frac{7}{3} > 1$ 2

SCORE: _____ / 20 PTS

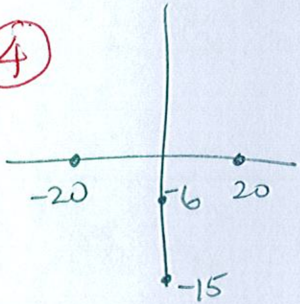
[a] What is the shape of the graph of the equation ?

HYPERBOLA 2

[b] Find the **rectangular** coordinates of the endpoints of **all** latera recta.

θ	r
0	20
$\frac{\pi}{2}$	-15
π	20
$\frac{3\pi}{2}$	6

4



OTHER FOCUS @ $-6 + -15 = -21$ 4

ENDPOINTS OF LR

$(\pm 20, 0)$ 3

$(\pm 20, -21)$ 3

Find all angles in the interval $[0, 2\pi]$ at which the graph of $r = 1 + 2\sin 2\theta$ goes through the pole. **SCORE: _____ / 15 PTS**

3 $\sin 2\theta = -\frac{1}{2}$ $\quad\quad \theta \in [0, 2\pi]$

$2\theta = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}$ $\quad\quad 2\theta \in [0, 4\pi]$

8

$\theta = \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$

4

Write and **prove** a formula for $\cosh 2x$ that uses both $\sinh x$ and $\cosh x$.

SCORE: _____ / 20 PTS

Your proof must use the exponential definitions of the hyperbolic functions.

$$\cosh^2 x + \sinh^2 x = \left(\frac{e^x + e^{-x}}{2}\right)^2 + \left(\frac{e^x - e^{-x}}{2}\right)^2$$ 2

4

$$= \frac{e^{2x} + \cancel{2} + e^{-2x} + e^{2x} - \cancel{2} + e^{-2x}}{4}$$ 4

$$= \frac{2e^{2x} + 2e^{-2x}}{4} = \frac{e^{2x} + e^{-2x}}{2} = \cosh 2x$$ 2

Using that formula, write and prove a formula for $\cosh 2x$ that uses only $\sinh x$.

Your proof must use the hyperbolic identity that resembles the trigonometric Pythagorean identity.

$$\cosh^2 x - \sinh^2 x = 1 \longrightarrow \cosh^2 x = 1 + \sinh^2 x$$ 2

3

$$\cosh x = 1 + \sinh^2 x + \sinh^2 x$$

$$= 1 + 2\sinh^2 x$$ 3

AJ and BJ were working on their polar graphing partner quiz. **SCORE: _____ / 20 PTS**

On the question about the polar equation $r = 2 + 2\sin 3\theta$, they determined correctly that the symmetry tests $(r, \pi + \theta)$, $(-r, \pi - \theta)$ and $(r, -\theta)$ do **NOT** indicate that the graph is symmetric.

POLE POLARAXIS POLARAXIS

[a] **Using their results, along with the tests and shortcuts shown in lecture**, test if the graph is symmetric over the pole, the polar axis and/or $\theta = \frac{\pi}{2}$. State your conclusions in the table. **NOTE: Run as FEW tests as needed to prove your answers are correct.**

POLE: $-r = 2 + 2\sin 3\theta$ (3)

$r = -2 - 2\sin 3\theta$ × (3)

$\theta = \frac{\pi}{2}$: $r = 2 + 2\sin 3(\pi - \theta)$ (3)

$r = 2 + 2\sin(3\pi - 3\theta)$

$r = 2 + 2[\cancel{\sin 3\pi}^{0}\cos 3\theta - \cancel{\cos 3\pi}^{-1}\sin 3\theta]$

$r = 2 + 2\sin 3\theta$ ✓ (3)

Type of symmetry	Conclusion
Over the pole	NO CONCLUSION
Over the polar axis	NO CONCLUSION
Over $\theta = \frac{\pi}{2}$	SYMMETRIC

(4)

[b] Based on the results of part [a], what is the minimum interval of the graph you need to plot (before using reflections to draw the rest of the graph) ?

$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (4)

Rewrite $\operatorname{csch}(-2\ln x)$ in terms of exponential functions and simplify. **SCORE: _____ / 10 PTS**

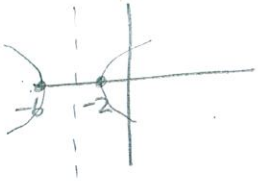
$$\underbrace{\frac{2}{e^{-2\ln x} - e^{2\ln x}}}_{4} = \underbrace{\frac{2}{x^{-2} - x^{2}}}_{3} \cdot \frac{x^2}{x^2} = \underbrace{\frac{2x^2}{1 - x^4}}_{3}$$

A hyperbola has a focus at the pole and vertices with **rectangular** co-ordinates $(-6, 0)$ and $(-2, 0)$. **SCORE: _____ / 20 PTS**

[a] Find polar co-ordinates for the vertices, using positive values of r and θ.

$(6, \pi), (2, \pi)$ 4

[b] Find the **polar** equation of the hyperbola.



$r = \frac{ep}{1 - e\cos\theta}$ 4

$2 = \frac{ep}{1 - e\cos\pi}$ $\quad -6 = \frac{ep}{1 - e\cos 0}$

2 $\quad 2 = \frac{ep}{1+e}$ $\quad -6 = \frac{ep}{1-e}$ 4

$ep = 2 + 2e$ $\quad ep = -6 + 6e$

$2 + 2e = -6 + 6e$ 2

$8 = 4e$

1 $e = 2$

$2p = 2 + 2(2) = 6$

1 $p = 3$

$r = \frac{2(3)}{1 - 2\cos\theta}$

$r = \frac{6}{1 - 2\cos\theta}$ 2

Find the logarithmic formula for $\tanh^{-1} x$ by solving $x = \tanh y$ for y using the exponential definition and an algebraic substitution $z = e^y$.

SCORE: _____ / 20 PTS

$$x = \frac{e^y - e^{-y}}{e^y + e^{-y}} = \frac{z - \frac{1}{z}}{z + \frac{1}{z}} \cdot \frac{z}{z} = \frac{z^2 - 1}{z^2 + 1}$$ 2 5

$$x(z^2+1) = z^2 - 1$$

$$xz^2 + x = z^2 - 1$$ 2

$$1 + x = z^2 - xz^2 = (1-x)z^2$$ 2

$$z^2 = \frac{1+x}{1-x}$$ 2

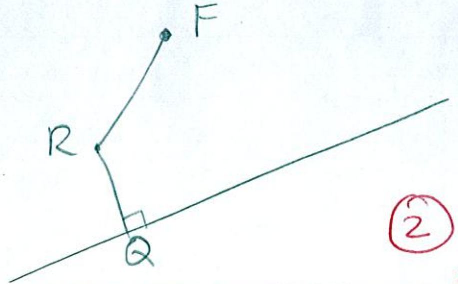
$$z = \pm\sqrt{\frac{1+x}{1-x}} \quad \text{BUT } z = e^y > 0$$ 2

$$e^y = \sqrt{\frac{1+x}{1-x}}$$ 2

$$y = \ln\sqrt{\frac{1+x}{1-x}} = \frac{1}{2}\ln\frac{1+x}{1-x}$$ 3

A drinking fountain is 6 feet from the wall of a school building. A rabbit is running on the school grounds, so that it is always one third as far from the wall as it is from the fountain. What is the shape of the rabbit's path ? **SCORE: _____ / 10 PTS**

<u>**Draw a diagram and write algebraic equations involving distances to justify your answer.**</u>



2

$RQ = \frac{1}{3} RF$ 3

$3 = \frac{RF}{RQ} = e$ 3

HYPERBOLA 2

Convert the polar equation $r = 1 + \sin 2\theta$ to rectangular and simplify. **SCORE: _____ / 15 PTS**

$r = 1 + \underbrace{2\sin\theta\cos\theta}$ (4)

$r = 1 + \underbrace{2\left(\frac{y}{r}\right)\left(\frac{x}{r}\right)}$ OR $\underbrace{r^3 = r^2 + 2(r\sin\theta)(r\cos\theta)}$

$r = 1 + \frac{2xy}{r^2}$ (3)

$\underbrace{r^3 = r^2 + 2xy}$ (3)

$\underbrace{\left(\sqrt{x^2+y^2}\right)^3 = x^2+y^2+2xy}$ (3)

$(x^2+y^2)^{\frac{3}{2}} = (x+y)^2$

$\underbrace{(x^2+y^2)^3 = (x+y)^4}$

(2)